Opening the 'black box': Understanding the Equation of IndoTERM

Day 1 Section 3

Adopted from term.ppt by Mark Horridge

Persamaan dalam CGE

- market-clearing conditions for commodities and primary factors;
- producers' demands for produced inputs and primary factors;
- final demands (investment, household, export and government);
- the relationship of prices to supply costs and taxes; and
- macroeconomic variables and price indices.

Linearisasi

- Tujuan:

  mempermudah pemecahan numerik dari persamaan dalam model CGE

- Kesepakatan Umum
  - Huruf KAPITAL: nilai original
  - Huruf kecil: persentase perubahan dari nilai original (nilai marginal)
  - $A*a=100\Delta A$ atau $A*a=100dA$

How to derive percent-change equations

a levels equation, for example,

$$Y = X^2 + Z,$$

is turned into percent-change form by first taking total differentials:

$$dY = 2XdX + dZ.$$

Percent changes x, y, and z are defined *via*:

$$y = 100\frac{dY}{Y} \quad \text{or} \quad dY = \frac{Yy}{100},$$

$$\text{similarly} \quad dX = \frac{Xx}{100} \quad \text{and} \quad dZ = \frac{Zz}{100}.$$

Thus our sample equation becomes:

$$\frac{Yy}{100} = 2X\frac{Xx}{100} + \frac{Zz}{100}, \qquad \text{or} \qquad Yy = 2X^2x + Zz.$$

$y = 2x + (Z/Y)z$

Linearization

- Capital letter, X, is level variable, small case letter, x, is percentage change variable, or

$$x = \frac{dX}{X}100$$

- Multiplication

$$\begin{aligned}
Y &= X_1 X_2 \\
dY &= \frac{\partial Y}{\partial X_1} dX_1 + \frac{\partial Y}{\partial X_2} dX_2 \\
dY &= X_2 dX_1 + X_1 dX_2 \\
\frac{dY}{Y}100 &= \frac{X_2 dX_1}{X_1 X_2}100 + \frac{X_1 dX_2}{X_1 X_2}100 \\
y &= x_1 + x_2
\end{aligned}$$

More generally,

$$Y = \prod_i X_i \Rightarrow y = \sum_i x_i$$

Linearization (Addition)

Note that we can write $dX = \frac{Xx}{100}$.

$$\begin{aligned}
Y &= X_1 + X_2 \\
dY &= \frac{\partial Y}{\partial X_1} dX_1 + \frac{\partial Y}{\partial X_2} dX_2 \\
dY &= dX_1 + dX_2 \\
\frac{Yy}{100} &= \frac{X_1 x_1}{100} + \frac{X_2 x_2}{100} \\
Yy &= X_1 x_1 + X_2 x_2, \text{ or} \\
y &= \frac{X_1}{Y} x_1 + \frac{X_2}{Y} x_2 \\
y &= S_1 x_1 + S_2 x_2
\end{aligned}$$

where S_1 and S_2 are share of X_1 and X_2 respectively. More generally

$$Y = \sum_i X_i \Rightarrow Yy = \sum_i X_i x_i, \text{ or } y = \sum_i S_i x_i$$

Linearization (Power)

$$
\begin{aligned}
Y &= X_1^{\alpha_1} X_2^{\alpha_2} \\
dY &= \frac{\partial Y}{\partial X_1} dX_1 + \frac{\partial Y}{\partial X_2} dX_2 \\
&= \alpha_1 X^{\alpha_1 - 1} X_2^{\alpha_1} \cdot dX_1 + \alpha_2 X_1^{\alpha_1} X_2^{\alpha_2 - 1} \cdot dX_2 \\
&= \alpha_1 \frac{Y}{X_1} dX_1 + \alpha_2 \frac{Y}{X_2} dX_2 \\
\frac{dY}{Y} 100 &= \alpha_1 \frac{Y}{Y} \frac{dX_1}{X_1} 100 + \alpha_2 \frac{Y}{Y} \frac{dX_2}{X_2} 100 \\
y &= \alpha_1 x_1 + \alpha_2 x_2
\end{aligned}
$$

More generally

$$Y = \prod_i X_i^{\alpha_i} \Rightarrow y = \sum \alpha_i x_i$$

3.4.2 Rule of Linearization

Rule 1 The linearized form of a constant is zero. $Y = \alpha \rightarrow y = 0$

Rule 2 $Y = X \rightarrow y = x$

Rule 3 The linearized form of multiplication is additive, or $Y = \prod_i X_i \rightarrow y = \sum_i x_i$. Example, the value of transaction $Y = P \cdot Q \rightarrow y = p + q$.

Rule 4 The linearized form of division is substration, or $Y = \frac{X_1}{X_2} \rightarrow y = x_1 - x_2$.

Rule 5 The linearized form of addition is share-weighted addition, or $Y = \sum_i X_i \rightarrow Yy = \sum_i X_i x_i$, or $y = \sum S_i x_i$, where $S_i = \frac{X_i}{Y}$ share of X_i in Y. Example, market clearing equation for labor $L^S = \sum_i L_i$, labor demand sum over industry is equal to labor supply. Applying the rule, $L^S l^S = \sum_i L_i l_i$, multiplying by W, where W is the same over industry, $WL^S \cdot l^S = \sum_i WL_i \cdot l_i$.

Rule 6 The linearized form of substraction is share-weighted substraction, or $Y = X_1 - X_2 \rightarrow Yy = X_1 x_1 - X_2 x_2$

Rule 7 The linearized form of power, is multiplication, or $Y = X^\alpha \rightarrow y = \alpha x$.

Table 2 Patterns for Percent-Change Equations

Pattern	(A) Original or Levels Form	(B) Intermediate Form	(C) Percent-Change Form
1	$Y = 4$	$Yy = 4*0$	$y = 0$
2	$Y = X$	$Yy = Xx$	$y = x$
3	$Y = 3X$	$Yy = 3Xx$	$y = x$
4	$Y = XZ$	$Yy = XZx + XZz$	$y = x + z$ *or* $y = x + 100(X/Y)\Delta Z$
5	$Y = X/Z$	$Yy = (X/Z)x - (X/Z)z$	$y = x - z$ *or* $100(Z)\Delta Y = Xx - Xz$ *or* $100\Delta Y = Y(x - z)$
6	$X_1 = M/4P_1$	$X_1x_1 = (M/4P_1)m - (M/4P_1)p_1$	$x_1 = m - p_1$
7	$Y = X^3$	$Yy = X^3 3x$	$y = 3x$
8	$Y = X^\alpha$	$Yy = X^\alpha \alpha x$	$y = \alpha x$ (α assumed constant)
9	$Y = X + Z$	$Yy = Xx + Zz$	$y = S_x x + S_z z$ where $S_x = X/Y$, etc
10	$Y = X - Z$	$Yy = Xx - Zz$	$y = S_x x - S_z z$ *or* $100(\Delta Y) = Xx - Zz$
11	$PY = PX + PZ$	$PY(y+p) = PX(x+p) + PZ(z+p)$ *or* $PYy = PXx + PZz$	$y = S_x x + S_z z$ where $S_x = PX/PY$, etc
12	$Z = \Sigma X_i$	$Zz = \Sigma X_i x_i$ *or* $0 = \Sigma X_i(x_i - z)$	$z = \Sigma S_i x_i$ where $S_i = X_i/Z$
13	$XP = \Sigma X_i P_i$ (adding up values)	$XP(x+p) = \Sigma X_i P_i(x_i+p_i)$ *or* $V(x+p) = \Sigma V_i(x_i+p_i)$ *where* $V_i = P_i X_i$ and $V = \Sigma V_i$	$x+p = \Sigma S_i(x_i+p_i)$ where $S_i = V_i/V$
14	$X = \Sigma X_i$ where all X_i have same price P	$Xx = \Sigma X_i x_i$ *or* $PXx = \Sigma PX_i x_i$ *or* $Vx = \Sigma V_i x_i$ *where* $V_i = PX_i$ and $V = \Sigma V_i$	$x = \Sigma S_i x_i$ where $S_i = V_i/V$
15	$XP = \Sigma X_i P_i$ (price and quantity indices)	$V(x+p) = \Sigma V_i(x_i+p_i)$ *where* $V_i = P_i X_i$ and $V = \Sigma V_i$	$Vx = \Sigma V_i x_i$ *or* $0 = \Sigma V_i(x - x_i)$ $Vp = \Sigma V_i p_i$ *or* $0 = \Sigma V_i(p - p_i)$

INDOMINI

Industry demand X_S as raw material

$$X\_S(c,i) = AINT(c,i)X1TOT(i)$$

CONSTANT LEONTIEF PARAMETER

OUTPUT

Demand for composite-goods as raw material only depend on the amount of output of the industry

INDOTERM

Industry demand X_S as raw material

```
E_xint_s (all,c,COM)(all,i,IND)(all,d,DST)
  xint_s(c,i,d) = atot(i,d) + aint_s(c,i,d) + xtot(i,d)
```

aint_s(c,i,d): CONSTANT LEONTIEF PARAMETER

xtot(i,d): OUTPUT

Demand for composite-goods as raw material only depend on the amount of output of the industry

INDOMINI

Supplies meets Demands: MARKET CLEARING FOR Domestic COMMODITIES

$$X1TOT(c) = X0(c,"dom")$$

INDOTERM

Market Clearing

Total demand for commodity c produced in r

=

supply commodity c produced in r

```
Equation E_xtrad_d
(all,c,COM)(all,s,SRC)(all,r,ORG) ID01(TRADE_D(c,s,r))*xtrad_d(c,s,r) =
                                         sum{d,DST, TRADE(c,s,r,d)*xtrad(c,s,r,d)};
```

INDOMINI

Input Market in MINIMAL

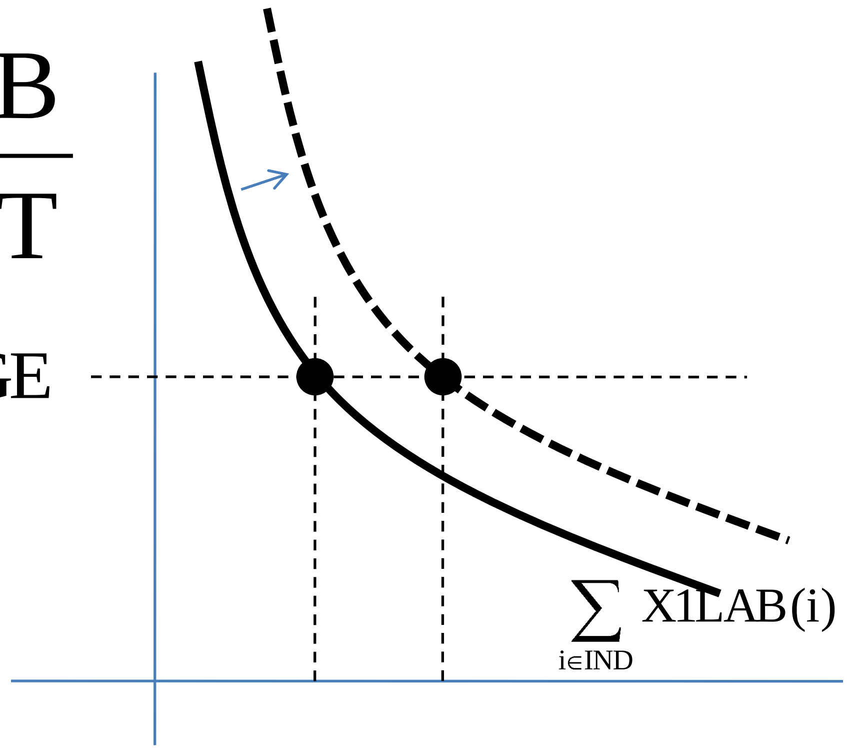
LABOR

$$EMPLOY = \sum_{i \in IND} X1LAB(i)$$

$$REALWAGE = \frac{P1LAB}{P3TOT}$$

REALWAGE

Exogenous



INDOTERM

Market Clearing for Labour

Total employment “o” di region “d”

```
E_xlab_i (all,o,OCC)(all,d,DST)
  xlab_i(o,d) = sum{i,IND, SLAB_I(i,o,d)*xlab(i,o,d)};
```

- Total employment di region “d”

```
E_xlab_io  (all,d,DST)
 LAB_IO(d)*xlab_io(d) = sum{o,OCC,LAB_I(o,d)*xlab_i(o,d)};
```

INDOTERM

Cost minimization: bottom level

$$\text{Minimize}: \text{P1LAB}\cdot\text{X1LAB}(i) + \text{P1CAP}(i)\cdot\text{X1CAP}(i)$$

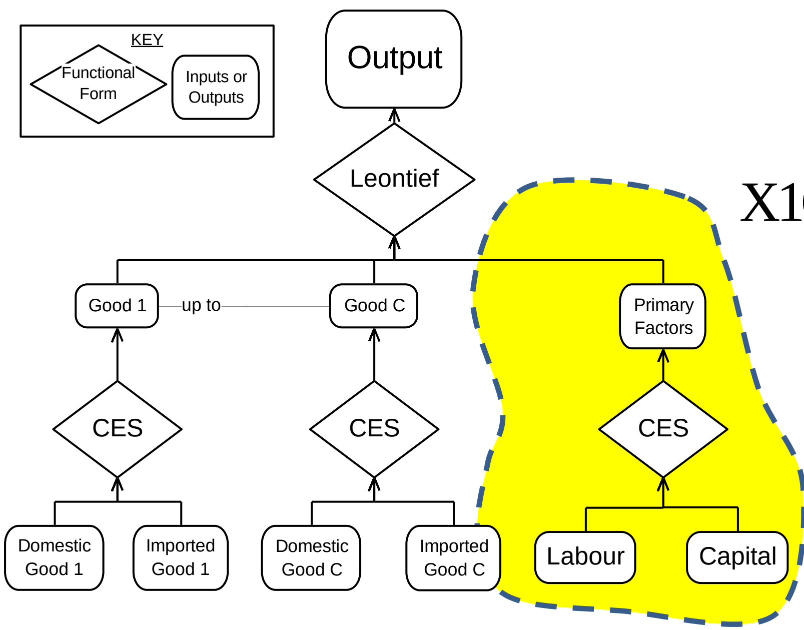
$$\text{Subject to}: \text{X1PRIM}(i) = \left[\phi_L \text{X1LAB}(i)^{-\rho} + \phi_K \text{X1CAP}(i)^{-\rho}\right]^{\frac{-1}{\rho}}$$

Solusi:

$$\text{X1LAB}(i) = \text{X1PRIM}(i)\cdot\phi_L^{\sigma_{PRIM}}\left(\frac{\text{P1LAB}}{\text{P1PRIM}(i)}\right)^{-\sigma_{PRIM}}$$

$$\text{X1CAP}(i) = \text{X1PRIM}(i)\cdot\phi_K^{\sigma_{PRIM}}\left(\frac{\text{P1CAP}(i)}{\text{P1PRIM}(i)}\right)^{-\sigma_{PRIM}}$$

$$\text{P1PRIM}(i)\cdot\text{X1PRIM}(i) = \text{P1LAB}\cdot\text{X1LAB}(i) + \text{P1CAP}(i)\cdot\text{X1CAP}(i)$$



INDOTERM

Skill Mix

Variable

(all,i,IND)(all,d,DST) plab_o(i,d) # Price of labour composite #;

(all,i,IND)(all,d,DST) xlab_o(i,d) # Effective labour input #;

Equation

E_xlab # Demand for labour by industry and skill group #

(all,i,IND)(all,o,OCC)(all,d,DST)

xlab(i,o ,d) = xlab_o(i ,d) - SIGMALAB(i)*[plab(i,o,d) - plab_o(i,d)];

E_plab_o # Price to each industry of labour composite #

(all,i,IND) (all,d,DST) ID01[LAB_O(i ,d)]*plab_o(i,d)

= sum{o,OCC,LAB(i,o,d)*plab(i,o,d)};

MEMORIZE $x_o = x_{average} - \underline{\sigma[p_o - p_{average}]}$

relative price term

CES PATTERN $p_{average} = \sum S_o . p_o$

INDOTERM

PUR_S(c,u,d)
ppur_s(c,u,d)
xhou_s(c,d)

PUR(c,s,u,d)
ppur(c,s,u,d)
xhou(c,s,d)

USE_U(c,s,d)
pdelivrd_r(c,s,d)
xtrad_r(c,s,d)

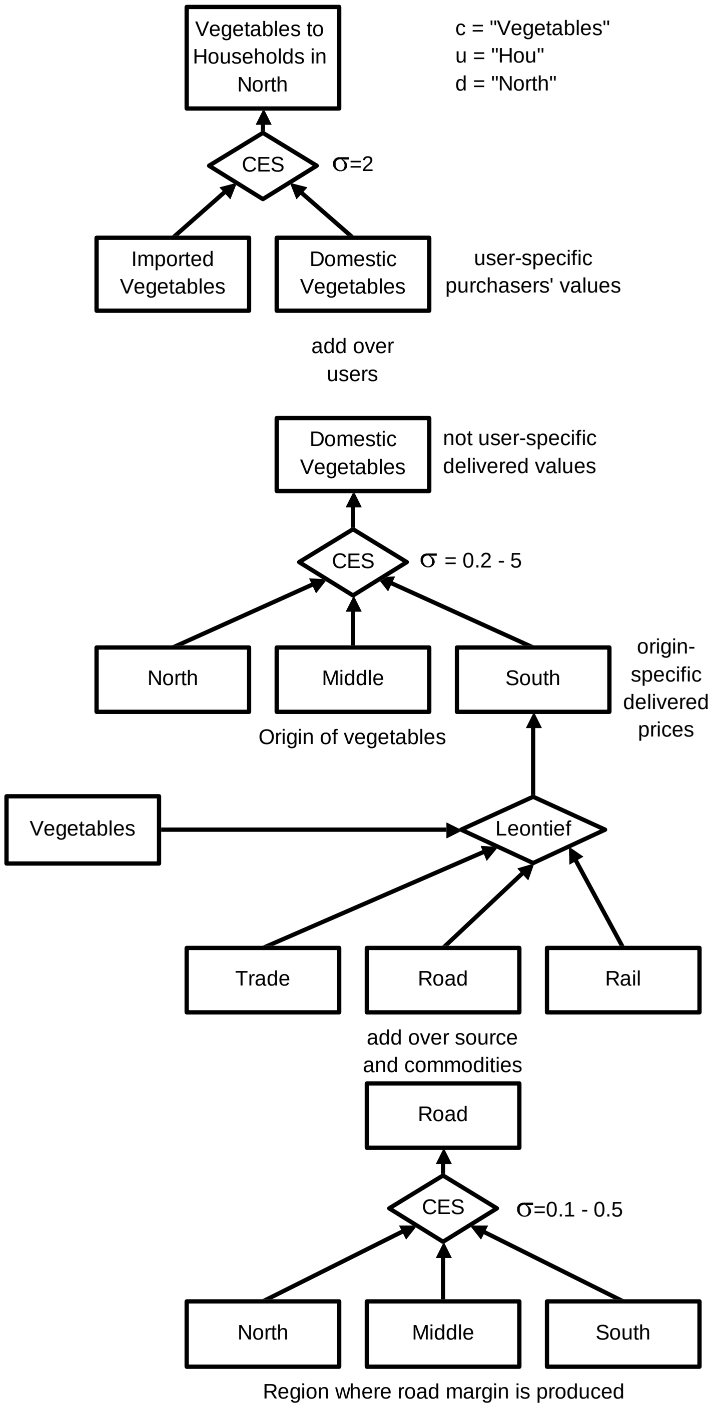
DELIVRD(c,s,r,d)
pdelivrdr(c,s,r,d)
xtrad(c,s,r,d)

TRADE(c,s,r,d)
pbasic(c,s,r)
xtrad(c,s,r,d)

TRADMAR(c,s,m,r,d)
psuppmar_p(m,r,d)
xtradmar(c,s,m,r,d)

TRADMAR_CS(m,r,d)
psuppmar_p(m,r,d)
xtradmar_cs(m,r,d)

SUPPMAR(m,r,d,p)
pdom(m,p)
xsuppmar(m,r,d,p)



INDOTERM

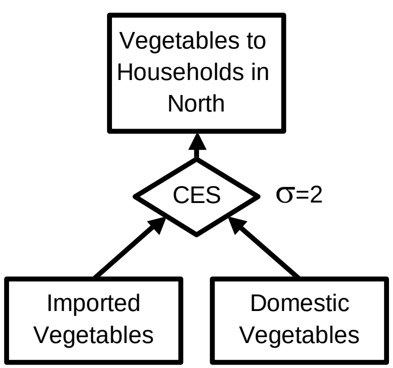
Intermediate Sourcing

Variable
(all,c,COM)(all,u,USR)(all,d,DST) ppur_s(c,u,d) *# User prices, average over s#*;
(all,c,COM)(all,i,IND)(all,d,DST)
    xint_s(c,i,d) *# Industry demands for dom/imp composite #*;
Equation
 E_ppur_s
(all,c,COM)(all,u,USR)(all,d,DST)
    ppur_s(c,u,d) = sum{s,SRC,SRCSHR(c,s,u,d)*ppur(c,s,u,d)};

E_xint (all,c,COM)(all,s,SRC)(all,i,IND)(all,d,DST)
xint(c,s,i,d) = xint_s(c,i,d) - SIGMADOMIMP(c)*[ppur(c,s,i,d)-ppur_s(c,i,d)];

$x_s = x_{average} - s[p_s - p_{average}]$

PUR_S(c,u,d)
ppur_s(c,u,d)
xhou_s(c,d)



c = "Vegetables"
u = "Hou"
d = "North"

user-specific purchasers' values

$$p_{average} = \sum S_s.[p_s]$$

PUR(c,s,u,d)
ppur(c,s,u,d)
xhou(c,s,d)

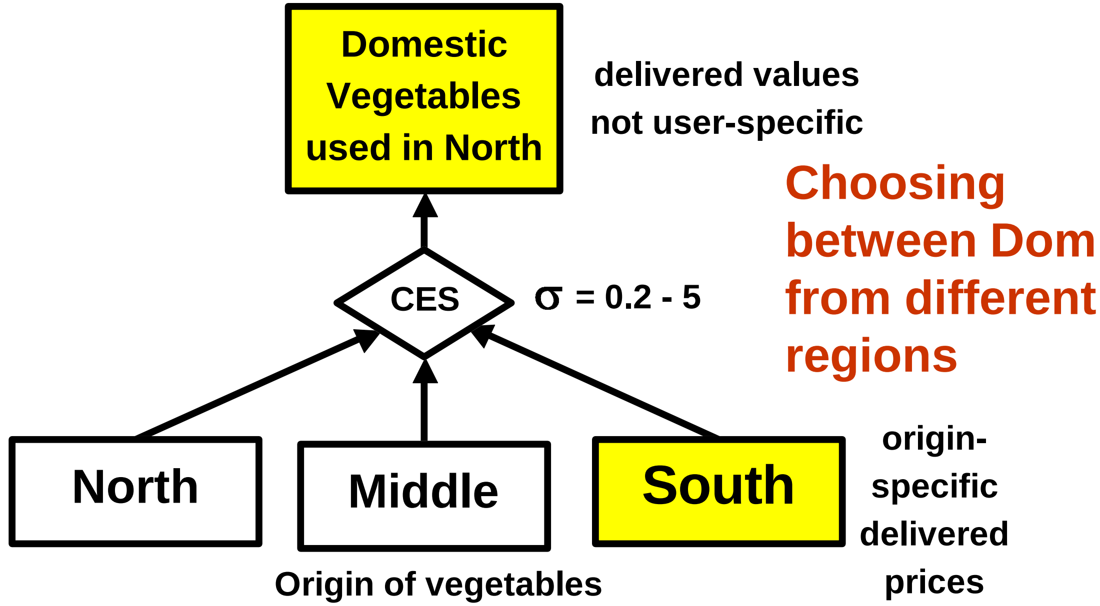
SAME FOR HOU AND INV USERS

INDOTERM

Regional sourcing of goods

USE_U(c,s,d)
puse(c,s,d)
xtrad_r(c,s,d)

DELIVRD(c,s,r,d)
pdelivrdr(c,s,r,d)
xtrad(c,s,r,d)



Equation E_puse *# Delivered price of regional composite good c,s to d #*

```
(all,c,COM)(all,s,SRC)(all,d,DST)
 ID01(DELIVRD_R(c,s,d))*puse(c,s,d) ! CES price index !
        = sum{r,ORG,DELIVRD(c,s,r,d)*[pdelivrd(c,s,r,d)+atrad(c,s,r,d)]};
```

Equation E_xtrad *# CES between goods from different regions #*

```
(all,c,COM)(all,s,SRC)(all,r,ORG)(all,d,DST)
 xtrad(c,s,r,d) - atrad(c,s,r,d) = xuse(c,s,d)
        - SIGMADOMDOM(c)*[pdelivrd(c,s,r,d)+atrad(c,s,r,d)-puse(c,s,d)];
```