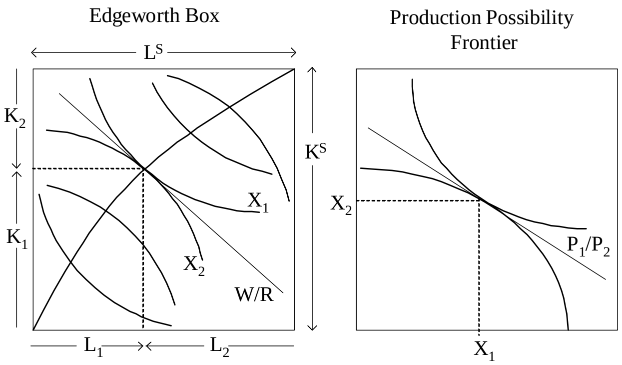
A 2 sectors, 2 factors General Equilibrium Model

Outline

- Diagramatic exposition
- Mathematical exposition
- Deriving closed-form solution
- Calibration
- Numerical solution: Group exercise
- Simulations
- General representation
- 7-steps to become CGE modellers
- Gempack exercises

Diagramatic exposition



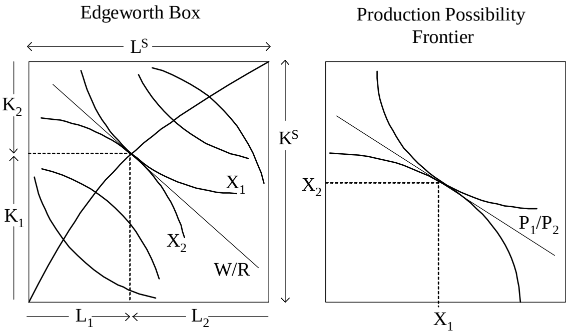
- Find K_1, L_1, K_2, L_2, the optimum allocation of factor endowment K^S, L^S.
- Endogenous variables: $K_1, L_1, K_2, L_2, X_1, X_2, P_1, P_2, W, R$
- Exogenous variables: K^S, L^S

Mathematically

- What we know already
  - TECHNOLOGY: Isoquant map of industry 1 and 2 $X_i = X_i(K_i, L_i)$
    - Suppose it is Cobb-Douglass $X_1 = K_1^{\alpha} L_1^{1-\alpha}$, $X_2 = K_2^{\beta} L_2^{1-\beta}$
  - PREFERENCE: Indifference curve of consumers $U = U(X_1, X_2)$
    - Suppose it is Cobb-Douglass $U = X_1^{\gamma} X_2^{1-\gamma}$
  - RESOURCE CONSTRAINT: Exogenous variables K^S, L^S.
- What we want to know
  - Quantity variables: the optimum K_1, L_1, K_2, L_2 and X_1, X_2
  - Price variables: the equilibrium P_1, P_2, W, R
- How? Draw Edgworth Box and PPF/Indifference Curve (If you can)
- Solve it mathematically, assuming that
  - Firms is minimizing costs and household maximizing utility in a competitive market
  - This is the competitive solution $\approx$ Social planner solution (Invisible hand, 1st theorem of Welfare Economics)

The 1st Theorem of Welfare Economics

Competitive Solution is equal to social planner solution



- A representative social planner "Mr Knows Everything", order the allocation of resource.
- Firms and households don't care of others, maximizing profit and utility in a competitive market.
- Both solutions are pareto optimum (Arrow and Debreu).

Deriving solution

- Producer's problem:
  - Producer 1: min $WL_1 + RK_1$ subject to $X_1 = K_1^{\alpha} L_1^{1-\alpha}$
  - Producer 2: min $WL_2 + RK_2$ subject to $X_2 = K_2^{\beta} L_2^{1-\beta}$
- Consumer's problem:
  - max $U = X_1^{\gamma} X_2^{1-\gamma}$ subject to $Y = P_1 X_1 + P_2 X_2$
- Solution to producer's problem (see previous module):
  - Producer 1: $K_1 = \frac{\alpha c_1 X_1}{R}; L_1 = \frac{(1-\alpha) c_1 X_1}{W}$
  - Producer 2: $K_2 = \frac{\beta c_2 X_2}{R}; L_2 = \frac{(1-\beta) c_2 X_2}{W}$
  - Competitive market implies zero profit: $P_1 = c_1; P_2 = c_2$
- Solution to consumer's problem:
  - $X_1 = \gamma \frac{Y}{P_1}; X_2 = (1-\gamma)\frac{Y}{P_2} \Rightarrow \frac{P_1 X_1}{P_2 X_2} = \frac{\gamma}{1-\gamma} \Rightarrow P_2 X_2 = \frac{1-\gamma}{\gamma} P_1 X_1$

Deriving solution (cont.)

- Producer 1:
  $K_1 = \frac{\alpha c_1 X_1}{R} \Rightarrow R = \frac{\alpha c_1 X_1}{K_1}; L_1 = \frac{(1-\alpha) c_1 X_1}{W} \Rightarrow W = \frac{(1-\alpha) c_1 X_1}{L_1}$
- Producer 2:
  $K_2 = \frac{\beta c_2 X_2}{R} \Rightarrow R = \frac{\beta c_2 X_2}{K_2}; L_2 = \frac{(1-\beta) c_2 X_2}{W} \Rightarrow W = \frac{(1-\beta) c_2 X_2}{L_2}$
- Equate R and use zero profit:
  $\frac{\alpha c_1 X_1}{K_1} = \frac{\beta c_2 X_2}{K_2} \Rightarrow \frac{\alpha P_1 X_1}{K_1} = \frac{\beta P_2 X_2}{K_2}$
- Substitute this: $P_2 X_2 = \frac{1-\gamma}{\gamma} P_1 X_1$
  $\frac{\alpha P_1 X_1}{K_1} = \frac{\beta \frac{1-\gamma}{\gamma} P_1 X_1}{K_2} \Rightarrow \frac{\alpha}{K_1} = \frac{\beta(1-\gamma)}{\gamma K_2}$
- Equate W and use zero profit:
  $\frac{(1-\alpha) c_1 X_1}{L_1} = \frac{(1-\beta) c_2 X_2}{L_2} \Rightarrow \frac{(1-\alpha) P_1 X_1}{L_1} = \frac{(1-\beta) P_2 X_2}{L_2}$
- Substitute this: $P_2 X_2 = \frac{1-\gamma}{\gamma} P_1 X_1$
  $\frac{(1-\alpha) P_1 X_1}{L_1} = \frac{(1-\beta) \frac{1-\gamma}{\gamma} P_1 X_1}{L_2} \Rightarrow \frac{1-\alpha}{L_1} = \frac{(1-\beta)(1-\gamma)}{\gamma L_2}$

- We have found:

$$\frac{\alpha}{K_1} = \frac{\beta(1-\gamma)}{\gamma K_2} \text{ and } \frac{1-\alpha}{L_1} = \frac{(1-\beta)(1-\gamma)}{\gamma L_2}$$

- But we have resource constraint equations:

$$K^S = K_1 + K_2 \text{ and } L^S = L_1 + L_2$$

- Solve for K_1: $\frac{\alpha}{K_1} = \frac{\beta(1-\gamma)}{\gamma(K^S - K_1)} \Rightarrow \alpha\gamma K^S - \alpha\gamma K_1 = \beta(1-\gamma) K_1$
  $(\beta(1-\gamma) + \alpha\gamma) K_1 = \alpha\gamma K^S \Rightarrow K_1 = \frac{\alpha\gamma}{\beta(1-\gamma)+\alpha\gamma} K^S$
- Solve for L_1:
  $\frac{1-\alpha}{L_1} = \frac{(1-\beta)(1-\gamma)}{\gamma(L^S - L_1)} \Rightarrow (1-\alpha)\gamma L^S - (1-\alpha)\gamma L_1 = (1-\beta)(1-\gamma) L_1$
  $((1-\beta)(1-\gamma) + (1-\alpha)\gamma) L_1 = (1-\alpha)\gamma L^S$
  $\Rightarrow L_1 = \frac{(1-\alpha)\gamma}{(1-\beta)(1-\gamma)+(1-\alpha)\gamma} L^S$

Deriving solutions (cont.)

- The optimum for real (or quantity) variables

$$
\begin{aligned}
K_1 &= \frac{\alpha\gamma}{\beta(1-\gamma)+\alpha\gamma}K^S \\
L_1 &= \frac{(1-\alpha)\gamma}{(1-\beta)(1-\gamma)+(1-\alpha)\gamma}L^S \\
K_2 &= \frac{\beta(1-\gamma)}{\beta(1-\gamma)+\alpha\gamma}K^S \\
L_2 &= \frac{(1-\beta)(1-\gamma)}{(1-\beta)(1-\gamma)+(1-\alpha)\gamma}L^S \\
X_1 &= K_1^{\alpha}L_1^{1-\alpha} \\
X_2 &= K_2^{\beta}L_2^{1-\beta}
\end{aligned}
$$

- They depend on (1) factor endowment, (2) technology, and (3) preference

Finding Solution to GE Model (cont.)

- Price variables
  - GE model can NOT find absolute price level. Why?
  - Nominal homogeneity. Doubling all price, do not change real variables (see prev. slide). No unique solution of prices, you can always find many sets of equilibrium prices.
  - GE model can only find relative prices, price of commodity relative to one numeraire (corn economy).
- The solution for prices, with P_1 as numeraire, $P_1 = 1$

$$
\begin{aligned}
P_1 &= 1 \\
P_2 &= \frac{(1-\gamma)}{\gamma}\frac{X_1}{X_2} \\
R &= \alpha\frac{X_1}{K_1} \\
W &= (1-\alpha)\frac{X_1}{L_1}
\end{aligned}
$$

Calibration

- Data (of the economy) represent initial equilibrium: *Important Assumption!!*
- What kind of data? For example: Input-Output Table

	Industry		Consumption	
	1	2		Total
1			P_1X_1	P_1X_1
2			P_2X_2	P_2X_2
Salary	WL_1	WL_2		WL^S
Surplus	RK_1	RK_2		RK^S
Total	P_1X_1	P_2X_2		Y

Calibration (cont.)

- I-O Table Indonesia (2000)

	Industry 1	Industry 2	Consumption	Total
1			347	347
2			953	953
Salary	68	340		408
Surplus	279	613		892
Total	347	953	1300	1300

- CGE model has to represent the data.
- If solved, it has to be able to reproduce this data (initial equilibrium).
- Calibration means "finding parameters of GE model (i.e. α, β, γ) that can reproduce the initial equilibrium exactly like this data".

Calibration (Example)

- From Input-Output Table, $WL_1 = 68.05$, $WL_2 = 340.13$, $RK_1 = 278.55$, and $RK_2 = 613.17$.
- Then, $P_1X_1 = WL_1 + RK_1 = 346.6$, and $P_2X_2 = WL_2 + RK_2 = 953.3$.
- Then, derived from first order conditions

$$\alpha = \frac{RK_1}{P_1X_1} = 0.804$$

$$\beta = \frac{RK_2}{P_2X_2} = 0.643$$

$$\gamma = \frac{P_1X_1}{P_1X_1 + P_2X_2} = 0.267$$

- Now, we have all the parameters!

Finding (numerical) Solutions

- K^S, L^S? We know that $RK^S = RK_1 + RK_2 = 891.72$, and $WL^S = WL_1 + WL_2 = 408.18$.
- Assume, that $W = 1$, and $R = 1$, so we can say that $K^S = 891.72$, and $L^S = 408.18$. Then,
- Exercise #1: We can complete the following (Try to do it yourself):
  $K_1 = \frac{\alpha\gamma}{\beta(1-\gamma)+\alpha\gamma} K^S =$
  $L_1 = \frac{(1-\alpha)\gamma}{(1-\beta)(1-\gamma)+(1-\alpha)\gamma} L^S =$
  $K_2 = \frac{\beta(1-\gamma)}{\beta(1-\gamma)+\alpha\gamma} K^S =$
  $L_2 = \frac{(1-\beta)(1-\gamma)}{(1-\beta)(1-\gamma)+(1-\alpha)\gamma} L^S =$
  $X_1 = K_1^{\alpha} L_1^{1-\alpha} =$, and $X_2 = K_2^{\beta} L_2^{1-\beta} =$

Numerical solution (continued)

Complete the following:

$$
\begin{aligned}
P_1 &= 1 \\
P_2 &= \frac{(1-\gamma)}{\gamma}\frac{X_1}{X_2} = \\
R &= \alpha\frac{X_1}{K_1} = \\
W &= (1-\alpha)\,\frac{X_1}{L_1} =
\end{aligned}
$$

- Try to repeat the whole exercise using different assumed value of W and R! What can you conclude?

Interpretation of GE (initial) solutions

- Interpretation of prices. $P_1 = 1, P_2 = 1.17, W = 0.61, R = 0.61$. Answer the following questions?
  - Can we find prices in Rupiahs? Why?
  - Can we say $P_2 > P_1$ or $P_1 > W$? Why?
  - Can we say $W = R$? Why?
- Interpretation of quantity. $K_1 = 278, K_2 = 613, L_1 = 68.05, L_2 = 340.13, X_1 = 211.22, X_2 = 497$.
  - What is the unit of labor? Man-Hour?
  - What is the unit of K?
  - Can we say $X_1 < X_2$? Why?
  - Can we say $K_1 < K_2$ or $L_1 < L_2$?
- The value of CGE initial solutions depend on what is the numeraire (for prices), and depend on the initial value of W and R that we "guess" (for quantities).

CGE is for Comparative Static Analysis

- If those solutions tell nothing, what is CGE for???
- Here is the biggest magic: *CGE model will tell you everything in comparative static analysis!*
- Comparative static: change the exogenous variables, K^S, L^S (simulation), solve again, and compare it with initial solution.
- The (percentage) change of all the variables are independent on initial value we "guess", and are comparable among them

CGE is for Comparative Static Analysis (cont.)

Effect of change in factor endowment

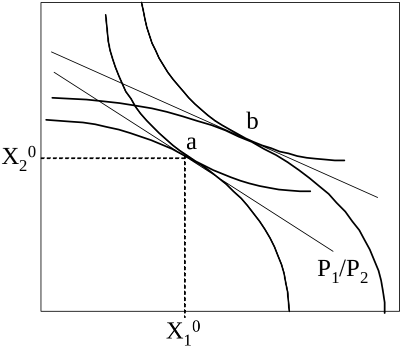


Figure 2.1.a

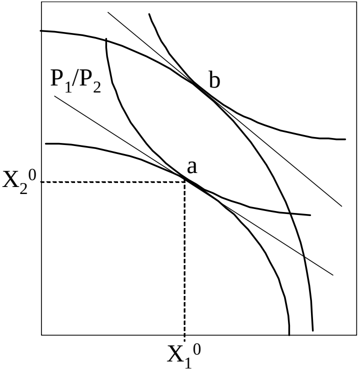


Figure 2.1.b

Exercise #2

- Increase L^S by 20%, complete the table below:

Initial	After	%	Initial	After	%
$K_1 = 278.55$	$K_1 =$		$R = 0.61$	$R =$	
$K_2 = 613.17$	$K_2 =$				
$L_1 = 68.05$	$L_1 =$		$W = 0.61$	$W =$	
$L_2 = 340.13$	$L_2 =$				
$X_1 = 211.22$	$X_1 =$		$P_1 = 1.00$	$P_1 =$	
$X_2 = 496.9$	$X_2 =$		$P_2 = 1.17$	$P_2 =$	

Exercise (cont)

- Repeat exercise #1, but change the assumed W and R by any amount you want (Remember that in previous exercise, we assume $W = R = 1$). Complete the following table!

Initial	After	%	Initial	After	%
$K_1 =$	$K_1 =$		$R =$	$R =$	
$K_2 =$	$K_2 =$				
$L_1 =$	$L_1 =$		$W =$	$W =$	
$L_2 =$	$L_2 =$				
$X_1 =$	$X_1 =$		$P_1 =$	$P_1 =$	
$X_2 =$	$X_2 =$		$P_2 =$	$P_2 =$	

- Discuss what you learn from changing the initial guess value of W and R.
- Increase P_1 by 50% and discuss its effect?
- Increase both K_S and L_S by 50% and discuss its effect?

Homogeneity in CGE models

- You just learned two important properties of any standard CGE models.
- *Nominal homogeneity or price homogeneity*
  - If you increase numeraire by the x% then all prices and nominal variables will increase by x% but all real or quantity variabels will remain unchanged.
- *Real homogenity or constant returns to scale*
  - If you increase all exogenous quantity variables then by x% then all real or quantity and nominal variables will increase by x% but all prices will remain unchanged.
- It is important to always test any CGE model for both homogenity tests.

More General Representation of the GE Model

- Firm i's problem, where $i \in 1, 2$

$$\min_{K_i, L_i} W \cdot L_i + R \cdot K_i \quad \text{s.t.} \quad X_i = X_i\left(K_i, L_i\right)$$

and this will produce demand for factors

$$K_i = K_i\left(W, R, X_i\right) \tag{1}$$

$$L_i = L_i\left(W, R, X_i\right) \tag{2}$$

- Household's problem is

$$\max_{X_i} U\left(Q_i\right) \text{ s.t} \sum P_i Q_i = Y.$$

where

$$Y = W \cdot L^S + R \cdot K^S \tag{3}$$

and this will produce demand for commodity

$$Q_i = Q_i\left(P_i, Y\right) \tag{4}$$

More General Representation of the GE Model (cont.)

- Market Clearing for commodities
  Demand have to be equal to supply at the market for goods

$$X_i = Q_i. \tag{5}$$

- Market clearing for factors
  In factor market, demands have to be equal to their supply

$$K^S = \sum_i K_i \tag{6}$$

$$L^S = \sum_i L_i. \tag{7}$$

- Equation 1 to 7, are more general representation of GE model (2 Factors, 2 Sectors, closed economy).

With our previous example:

- Technology: $X_i = K_i^{\alpha_i} L_i^{1-\alpha_i}$, Preference: $U = \prod_i Q_i^{\gamma_i}$
- Demand for inputs:

$$
\begin{aligned}
K_i &= \frac{\alpha_i c_i X_i}{R} && \text{(E1)} \\
L_i &= \frac{(1-\alpha_i)\, c_i X_i}{W} && \text{(E2)} \\
c_i &= \left(\frac{R}{\alpha_i}\right)^{\alpha_i} \left(\frac{W}{1-\alpha_i}\right)^{1-\alpha_i} && \text{(E3)}
\end{aligned}
$$

- Zero profit condition:

$$P_i = c_i, \text{ or } P_i X_i = R \cdot K_i + W \cdot L_i \qquad \text{(E4)}$$

With our previous example (continued)

- Demand for commodities:

$$Q_i = \gamma_i \frac{Y}{P_i} \tag{E5}$$

- Household income:

$$Y = W \cdot L^S + R \cdot K^S \tag{E6}$$

- Market clearing for inputs:

$$K^S = \sum_i K_i \tag{E7}$$

$$L^S = \sum_i L_i \tag{E8}$$

- Market clearing for commodities:

$$Q_i = X_i \tag{E9}$$

With our previous example (continued)

- From equation E1 to E9, and $i = 1, 2$, we have
  - 15 equations, to determine
  - 17 variables
    $K_1, L_1, X_1, K_2, L_2, X_2, Q_1, Q_2, c_1, c_2, P_1, P_2, W, R, Y, K^S, L^S$
- To close the system, by Walras' law one market clearing will be dropped. It gives now:
  - 14 equations, to determine
  - 14 variables, becuase K^S, L^S are exogenous, and one of the price, let's say P_1 will be used as numeraire.

With CES technology:

- Technology: $X_i = \left(\alpha_i K_i^{-\rho_i} + \beta_i L_i^{-\rho_i}\right)^{\frac{-1}{\rho_i}}$, Preference: $U = \prod_i Q_i^{\gamma_i}$
- Demand for inputs:

$$K_i = X_i \alpha_i^{\sigma_i} \left(\frac{R}{c_i}\right)^{-\sigma_i} \quad \text{(E1)}$$

$$L_i = X_i \beta_i^{\sigma_i} \left(\frac{W}{c_i}\right)^{-\sigma_i} \quad \text{(E2)}$$

$$c_i = \left(\alpha_i^{\sigma_i} R^{1-\sigma_i} + \beta_i^{\sigma_i} W^{1-\sigma_i}\right)^{\frac{1}{1-\sigma_i}} \quad \text{(E3)}$$

- Zero profit condition:

$$P_i = c_i, \text{ or } P_i X_i = R \cdot K_i + W \cdot L_i \quad \text{(E4)}$$

With our previous example (continued)

- Demand for commodities:

$$Q_i = \gamma_i \frac{Y}{P_i} \tag{E5}$$

- Household income:

$$Y = W \cdot L^S + R \cdot K^S \tag{E6}$$

- Market clearing for inputs:

$$K^S = \sum_i K_i \tag{E7}$$

$$L^S = \sum_i L_i \tag{E8}$$

- Market clearing for commodities:

$$Q_i = X_i \tag{E9}$$

7 Steps to become a CGE modeler with Gempack

1. Understand the theory (in equations)
2. Linearization
3. Get and prepare the data
4. Creating Tablo file
5. Creating database file
6. Creating simulation file
7. Simulation

Step 1. The Theory

$$
\begin{aligned}
K_i &= X_i \alpha_i^{\sigma_i} \left(\frac{R}{c_i}\right)^{-\sigma_i} && \text{DD for capital} \\
L_i &= X_i \beta_i^{\sigma_i} \left(\frac{W}{c_i}\right)^{-\sigma_i} && \text{DD for labor} \\
c_i &= \left(\alpha_i^{\sigma_i} R^{1-\sigma_i} + \beta_i^{\sigma_i} W^{1-\sigma_i}\right)^{\frac{1}{1-\sigma_i}} && \text{Unit cost} \\
P_i X_i &= W L_i + R K_i && \text{Zero profit conditions} \\
Q_i &= \gamma_i \frac{Y}{P_i} && \text{DD for commodities} \\
Y &= W L^S + R K^S && \text{Income} \\
X_i &= Q_i && \text{Market clearing (com)} \\
L^S &= \sum_i L_i && \text{Market clearing (lab)}
\end{aligned}
$$

Query: Why no market clearing for capital?

Step 2. Linearization

1. $k_i = x_i - \sigma_i (r - c_i)$
2. $l_i = x_i - \sigma_i (w - c_i)$
3. $c_i = S_i^L \cdot w + (1 - S_i^L) \cdot r$
4. $P_i X_i \cdot (p_i + x_i) = WL_i \cdot (w + l_i) + RK_i (r + k_i)$
5. $q_i = y - p_i$
6. $Y \cdot y = WL^S \cdot (w + l^S) + RK^S \cdot (r + k^S)$
7. $x_i = q_i$
8. $WL^S \cdot l^S = \sum_i WL_i \cdot l_i$

Step 3. Get and prepare the data

	Industry		Consumption	
	1	2		Total
1			347	347
2			953	953
Salary	68	340		408
Surplus	279	613		892
Total	347	953	1300	1300

- WL_i and RK_i are base coefficient.
- Other coefficient, $P_iX_i = WL_i + RK_i, WL^S = \sum_i WL_i$, $RK^S = \sum_i RK_i$, $Y = WL^S + RK^S$.

File structure

1. FILE statement. Declaring a 'logical' file name where your data will be located.
2. SET statement. Declaring set names, or subscript in the model.
3. VARIABLE statement. Declaring all the variables in the model.
4. COEFFICIENT statement. Declaring all the coefficient and parameter in the model.
5. READ statement. Asking the program to read the 'base' coefficient from database.
6. FORMULA statement. Calculation of other coefficient as a function of base coefficient.
7. UPDATE statement. Updating the base coefficient.
8. EQUATION statement. Listing all the model equations.

Step 5. Creating Database File

- Database file record the 'base' coefficient, here the coefficients are WL_i and RK_i.
- In GEMPACK the database file has the extension .har, and created and read by a GEMPAK program called VIEWHAR.

Step 6. Creating Simulation File

- Simulation file is a text file with extension .CMF where you put the closure and shock statement, and some other ancilliary information.
- *Closure* is a stement declaring which variables are endogenous, and which variables are exogenous that make sure that the number of endogenous variables is equal to the number of statement and also will tell us what variables that can be shocked.
- *Shock statement* is changing one or combination of exogenous variables (our policy question) to see its impact on all the endogenous variables.

Step 7. Simulation

Asking GEMPACK to implement our 6 steps.

Now, it is your first time for computer exercise with Gempack!!